Momentum distribution of Δ- isobar in closed shell nuclei.

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One Δ- isobar components of the wave function in closed shell nuclei are considered within the framework of the harmonic oscillator model. Conventional transition potential is the π- and ρ- exchange potential. On the basis of the Δ- isobar configuration wave function, the momentum distribution of the Δ- isobar is calculated for the light nuclei ^4He,^{16}O,^{12}C.

Introduction

Within the framework of the traditional non-relativistic theory of nuclei, the non-nucleon degrees of freedom are bound up with the effect of mutual polarization or deformation of bound nucleons in nuclei due to close collisions within nuclei [1- 4]. In terms of the wave functions of nuclei this effect can be expressed by the introduction of additional to purely nucleon, isobar configurations. These configurations describe such states of nuclei, in which the part of nucleons in nuclei are in excited states, as virtual isobars, which appear due to the collisions within nuclei as a result the transitions

$$N + N \rightarrow N + \Delta(\Delta + \Delta) \rightarrow N + N.$$

The admixture of probability of these exotic nuclear configurations is small because of the small nuclear density and big isobar-nucleon mass difference. These exotic Δ- isobars (internal Δ- isobars) are far off the mass-shell unlike Δ- isobars (external Δ- isobars), which are born in reactions of particles with nuclei and are essentially on the mass-shell [5]. There were attempts to find manifestations of the exotic Δ- isobar configurations in the ground state of light nuclei in a number of experiments, in reactions on nuclei with knocking out Δ-isobars preexisting in the target nuclei[6-10]. A theoretical description of these reactions within the framework of the impulse approximation requires knowledge of the momentum distribution of Δ- isobars considered as components of nucleus, together with nucleons. This momentum distribution differs from the momentum distribution of the external isobars and does not depend on kinematics and characteristics of particles in reactions with nuclei.

In this work, the estimation is made for the probability of the internal Δ- isobars presence in light nuclei, and the momentum distribution of the Δ- isobars in light nuclei with the closed shells is obtained.

Wave functions of isobar configurations in closed shell nuclei.

According to the approach developed in the works of Arenhövel et al. [5,11,12], nucleons bound in the nucleus, in addition to the spatial, spin, and isospin coordinates, are characterized also by the intrinsic coordinate. For completeness and to fix the notation, let us summarize the results and formulas given in [11]. The state vector of N nucleons is

$$\mid \alpha_1(1), ...\alpha_N(N) >=\mid \beta_1(1)n_1(1), ...\beta_N(N)n_N(N) > .$$

The indices $\beta_1, ...\beta_N$ refer to the usual, spin, and isospin space. They take the same series of values for all particles, i.e., $\beta_1 = \alpha, \beta, \gamma...; ...\beta_N = \alpha, \beta, \gamma....$ The indices $n_1(1), ...n_N(N)$

characterize the state vectors in the intrinsic space. The basis vectors $\mid m_\nu >$ in the intrinsic space are defined as follows

$$\hat{M} \mid m_\nu >= m_\nu \mid m_\nu >,$$

where m_ν is the eigenvalue of mass operator $\hat{M}$ in the intrinsic space, and ν =1, 2, 3,.... The value of $m_{\nu=1}$ corresponds to a mass of nucleon, $m_{\nu=2}$ corresponds to a mass of first excited state - Δ, and so on. The vectors $\mid m_\nu >$ form the infinite discrete basis in the intrinsic space. Each particle is characterized by its intrinsic index of state $n_i(i)(i = 1...N)$. Later on, it is taken that $\mid n_i(i) >=\mid m_{\nu=1} >, \mid m_{\nu=2} >$,...., i.e., states are pure states. For brevity, we enter the designation $\mid m_{\nu=1} >=\mid N >, \mid m_{\nu=2} >=\mid \Delta >$,.... For instance, the vector describing the system of N nucleons is written as $\mid N(1)...N(N) >$; if the first particle is in the state of isobar, but the rest are nucleons, the state vector is written as $\mid \Delta(1)...N(N) >$.

For the system of N particles, which can be in different intrinsic states, hamiltonian of system H acts on spatial, spin, isospin, and intrinsic coordinates. According to [11] hamiltonian H have form

$$H = \sum_{k=1}^{A}(T(k) + H_{in}(k)) + \sum_{i \neq k} V(i,k).$$

Here, T(k) is the kinetic energy operator of k- particle, $H_{in}(k)$ is the part connected with the intrinsic degrees of freedom, V(i,k) is the two-particle interaction. The operators T and V, unlike those of standard nuclear physics, depend also on the intrinsic degrees of freedom. The operators T and H_{in} are diagonal on the intrinsic degrees of freedom. Let us assume that

$$H_{in} = \hat{M} - \hat{I} M_N,$$

i.e., the vectors $\mid m_\nu >$ are eigenvectors of the operator H_{in} with the eigenvalue $(m_\nu - M_N)$. Thus, the following formulae take place

$$< m_{\nu'} \mid H_{in} \mid m_\nu >= (m_{\nu'} - M_N)\delta_{\nu'\nu},$$

$$< m_{\nu'} \mid T \mid m_\nu >= \frac{p^2}{2M_\nu}\delta_{\nu'\nu},$$

$$< m_{\nu'} m_{\mu'} \mid V \mid m_\nu m_\mu >= V_{m_{\nu'} m_{\mu'}, m_\nu m_\mu}.$$

The wave function of the system of N particles in the state $\mid \alpha_1(1), ...\alpha_N(N) >$ characterized also by the intrinsic coordinates is introduced as follows

$$\psi_{\alpha_1,...\alpha_N}(\vec{r}_1, \sigma_z(1), \tau_z(1), m_{\nu_1}, ...\vec{r}_N, \sigma_z(N), \tau_z(N), m_{\nu_N}) =$$

$$< \vec{r}_1, \sigma_z(1), \tau_z(1), m_{\nu 1}, ...\vec{r}_N, \sigma_z(N), \tau_z(N), m_{\nu_N} \mid \alpha_1(1), ...\alpha_N(N) > .$$

An eigenfunction $\Psi_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N; m'_{\nu 1}...m'_{\nu_N})$ of N particles of operator H with eigenvalues $E_{\beta_1,...\beta_N}$ is a superposition of the wave functions belonging to the different configurations

$$\Psi_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N; m'_{\nu 1}...m'_{\nu_N}) = \sum_{n'_1,...n'_N} A_{n'_1,...n'_N} \phi_{n'_1,...n'_N}(m'_{\nu 1}...m'_{\nu_N}) \cdot \psi^{n'_1,...n'_N}_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N).$$

For brevity, we enter the designation $\vec{r}$ for space, spin, and isospin coordinates. The part of the wave function of nucleus $A_{n'_1,...n'_N} \phi_{n'_1,...n'_N}(m'_{\nu 1}...m'_{\nu_N}) \cdot \psi^{n'_1,...n'_N}_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N)$ describes configuration of N particles in nucleus with quantum numbers of the intrinsic states $n'_1, ...n'_N$.

For instance, the nucleon part of the nuclear wave function is characterized by the values $n'_1 = N(1), ..., n'_N = N(N)$, one-isobar configuration have $n'_1 = \Delta(1), ..., n'_N = N(N)$ and so on. By definition, $\psi^{n'_1,...n'_N}_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N)$ is the wave function of space, spin, and isospin coordinates of the configuration of N particles with quantum numbers $n'_1, ...n'_N$. The wave function $\phi_{n'_1,...n'_N}(m_{\nu 1}...m_{\nu_N}) =< m_{\nu_1}(1)...m_{\nu_N}(N) \mid n'_1(1)...n'_N(N) >$ is the intrinsic wave function of N particles. The wave function $\psi^{n'_1,...n'_N}_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N)$ should be antisymmetric for particles in the same intrinsic state. The remaining antisymmetrization for particles in different intrinsic states is done by operator $A_{n'_1,...n'_N}$.

The wave functions $A_{n'_1,...n'_N}\phi_{n'_1,...n'_N}(m'_{\nu 1}...m'_{\nu_N}) \cdot \psi^{n'_1,...n'_N}_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N)$ satisfy the following Schrödinger equation

$$\sum_{m_{\nu_1}...m_{\nu_N};m_{\nu'_1}...m_{\nu'_N}} \phi_{n_1...n_N}(m_{\nu_1}...m_{\nu_N})(H(\vec{r}_1...\vec{r}_N) - E_{\beta_1,...\beta_N})_{m_{\nu_1}...m_{\nu_N};m_{\nu'_1}...m_{\nu'_N}}$$

$$A_{n_1,...n_N}\phi_{n_1,...n_N}(m'_{\nu 1}...m'_{\nu_N}) \cdot \psi^{n_1,...n_N}_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N) =$$

$$-\sum_{m_{\nu_1}...m_{\nu_N};m_{\nu'_1}...m_{\nu'_N}} \phi_{n_1...n_N}(m_{\nu_1}...m_{\nu_N})(V)_{m_{\nu_1}...m_{\nu_N};m_{\nu'_1}...m_{\nu'_N}}$$

$$\sum_{n'_1,...n'_N \neq n_1,...n_N} A_{n'_1,...n'_N}\phi_{n'_1,...n'_N}(m'_{\nu 1}...m'_{\nu_N}) \cdot \psi^{n'_1,...n'_N}_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N).$$

There is no hope of solving this set of equations, because of the right- hand side of this equation couples the various isobar configurations. In practice, the impulse approximation is used, according to which on the right- hand side of this equation only the terms with the wave functions of nucleon configuration are leaved. Besides, the interactions between isobars and between isobars themselves are neglected on the left- hand side of equation. This approximation leaves only one-isobar and two- isobar configurations.

The full wave function $\Psi_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N; m'_{\nu 1}...m'_{\nu_N})$ must be normalized to the unit. Let us consider a bilinear form

$$\int d1...dN\Psi^*{}_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N; m'_{\nu 1}...m'_{\nu_N})\Psi_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N; m'_{\nu 1}...m'_{\nu_N})$$

Here, d1...dN means an integral of the space variable and the summation over the spin, isospin, and intrinsic coordinates. Using the condition of orthogonality for the intrinsic wave functions

$$\sum_{m'_{\nu 1}...m'_{\nu_N}} \phi^*_{n_1,...n_N}(m'_{\nu 1}...m'_{\nu_N})\phi_{n'_1,...n'_N}(m'_{\nu 1}...m'_{\nu_N}) = \delta_{n_1 n'_1}...\delta_{n_N n'_N}$$

and $A^+ = A$, $A^2 = \sqrt{N!}A$, we obtain

$$\int d1...dN\Psi^*{}_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N; m'_{\nu 1}...m'_{\nu_N})\Psi_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N; m'_{\nu 1}...m'_{\nu_N}) =$$

$$\int d\vec{r}_1...d\vec{r}_N \sum_{n'_1,...n'_N} \psi^{*n'_1,n'_1...n'_N}_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N)\psi^{n'_1,n'_1,...n'_N}_{\beta_1,\beta_2,...\beta_N}(\vec{r}_1, ...\vec{r}_N).$$

The indices $\beta_1, ...\beta_N$ are different for the closed shell states . If the wave function of the nucleon configuration is the Slater determinant, then

$$\int d1...dN\Psi^*{}_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N; m'_{\nu 1}...m'_{\nu_N})\Psi_{\beta_1,...\beta_N}(\vec{r}_1, ...\vec{r}_N; m'_{\nu 1}...m'_{\nu_N}) =$$

$$(1+\sum_{n'_1\neq N,...n'_N}\int d\vec{r}_1...d\vec{r}_N \psi^{*}{}^{n'_1,...n'_N}_{\beta_1,...\beta_N}(\vec{r}_1,...\vec{r}_N)\psi^{n'_1,...n'_N}_{\beta_1,...\beta_N}(\vec{r}_1,...\vec{r}_N)).$$

It is necessary to introduce a normalization factor

$$C=\sqrt{\frac{1}{1+\sum_{n'_1\neq N,...n'_N} W_{n'_1\neq N,...n'_N}}}$$

to obtain the full wave function $\Psi_{\beta_1,...\beta_N}(\vec{r}_1,...\vec{r}_N;m'_{\nu 1}...m'_{\nu_N})$ normalized on 1. Here,

$$W_{n'_1\neq N,...n'_N}=\int d\vec{r}_1...d\vec{r}_N \psi^{*}{}^{n'_1\neq N,...n'_N}_{\beta_1,...\beta_N}(\vec{r}_1,...\vec{r}_N)\psi^{n'_1\neq N,...n'_N}_{\beta_1,...\beta_N}(\vec{r}_1,...\vec{r}_N).$$

The wave function of the space, spin, and isospin coordinates of the isobar configuration with quantum numbers $n'_1\neq N,...n'_N$ is then

$$\psi'{}^{n'_1\neq N,...n'_N}_{\beta_1,...\beta_N}(\vec{r}_1,...\vec{r}_N)=C\psi^{n'_1\neq N,...n'_N}_{\beta_1,...\beta_N}(\vec{r}_1,...\vec{r}_N),$$

where $\psi^{n'_1\neq N,...n'_N}_{\beta_1,...\beta_N}(\vec{r}_1,...\vec{r}_N)$ is a solution of the Schrödinger equation.

It is sufficiently to deal with a two-body wave function of the ΔN system to find a momentum distribution of the delta- isobar in the nucleus. Let us consider the only one delta- isobar configuration. We write an antisymmetrization operator as follows

$$A=\frac{1}{\sqrt{N}}(1-\sum_{k=2}^{N}P_{1k})\frac{1}{\sqrt{N-1}}(1-\sum_{k=3}^{N}P_{2k})A_{3N},$$

where A_{3N} is the antisymmetrization operator of N-2 particles. One may start from the two-body pair wave function expansion

$$A_{3N}\psi^{\Delta(1),N(2),...N(N)}_{\beta_1,...\beta_N}(\vec{r}_1,...\vec{r}_N)=\sum_{\kappa_3,...\kappa_N}\psi^{\Delta(1)N(2)}_{\beta_1,\beta_2;\beta_3...\beta_N,\kappa_3,...\kappa_N}(\vec{r}_1,\vec{r}_2)A_{3N}\psi^{N(3),...N(N)}_{\kappa_3,...\kappa_N}(\vec{r}_3,...\vec{r}_N).$$

Here, $A_{3N}\psi^{N(2),...N(N)}_{\kappa_3,...\kappa_N}(\vec{r}_3,...\vec{r}_N)$ forms a complete set of the antisymmetric eigenfunctions of the N-2- particles system with energies $E_{\kappa_3,...\kappa_N}$. Substituting this expansion in the Schrödinger equation , multiplying the right- hand side of the this equation on $A_{3N}\psi^{N(3),...N(N)}_{\alpha_3,...\alpha_N}(\vec{r}_3,...\vec{r}_N)$, and using the condition of the orthogonality, one obtains the equation for the wave functions of the ΔN system

$$\sqrt{\frac{1}{N(N-1)}}\sum_{m_{\nu_1}...m_{\nu_N};m_{\nu'_1}...m_{\nu'_N}}\phi_{\Delta(1),N(2)...N(N)}(m_{\nu_1}...m_{\nu_N})$$

$$(T(1)+T(2)+\Delta M_1-E_{\beta_1,...\beta_N}+E_{\alpha_3,...\alpha_N})_{m_{\nu_1}...m_{\nu_N};m_{\nu'_1}...m_{\nu'_N}}$$

$$\phi_{\Delta(1),N(2)...N(N)}(m'_{\nu 1}...m'_{\nu_N})\psi^{\Delta(1),N(2)}_{\beta_1,\beta_2;\beta_3...\beta_N,\alpha_3,...\alpha_N}(\vec{r}_1,\vec{r}_2)=$$

$$-\sum_{m_{\nu_1}...m_{\nu_N};m_{\nu'_1}...m_{\nu'_N}}\phi_{\Delta(1),N(2)...N(N)}(m_{\nu_1}...m_{\nu_N})(V_{12})_{m_{\nu_1}...m_{\nu_N};m_{\nu'_1}...m_{\nu'_N}}\phi_{N(1),...N(N)}(m'_{\nu 1}...m'_{\nu_N})$$

$$\int d\vec{r}_3,...d\vec{r}_N A_{3N}\psi^{N(3),...N(N)}_{\alpha_3,...\alpha_N}(\vec{r}_3,...\vec{r}_N)A_N\cdot\psi^{N(1),...N(N)}_{\beta_1,...\beta_N}(\vec{r}_1,...\vec{r}_N).$$

As the wave function of the ΔN system and the wave function $\psi^{\Delta(1),N(2)...N(N)}_{\beta_1,...\beta_N}(\vec{r_1},...\vec{r}_N)$ are normalized to the same constant, the solution of the Schrödinger equation for the bound ΔN system must be multiplied by the above mentioned constant C. Solving this equation for the ΔN system in the shell model with the ls- coupling, one can obtain

$$\psi^{\Delta(1)N(2)}_{\beta_1,\beta_2;\beta_3...\beta_N,\alpha_3,...\alpha_N}(\vec{r_1},\vec{r_2}) = -G_2^{\Delta(1),N(2)}(E_{\alpha\beta})$$

$$\sum_{m_{\nu_1}...m_{\nu_N};m_{\nu_1'}...m_{\nu_N'}} \phi_{\Delta(1),N(2)...N(N)}(m_{\nu_1}...m_{\nu_N})(V_{12})_{m_{\nu_1}...m_{\nu_N};m_{\nu_1'}...m_{\nu_N'}}\phi_{N(1),...N(N)}(m'_{\nu1}...m'_{\nu_N})$$

$$\sqrt{2}A_{12}\phi_{\beta_1}(\vec{r_1})\phi_{\beta_2}(\vec{r_2})\delta_{\alpha_3\beta_3}...\delta_{\alpha_N\beta_N}.$$

Here, $\phi_\beta(\beta=\{nlm_lsm_stm_t\})$ are single- particle states. This approach allows to use potentials, which depend from the relative coordinates of nucleons and isobars. The two- particle propagator $G_2^{\Delta(1),N(2)}(E_{\alpha\beta})$ is

$$G_2^{\Delta(1),N(2)}(E_{\alpha\beta}) = [\sum_{m_{\nu_1}...m_{\nu_N};m_{\nu_1'}...m_{\nu_N'}} \phi_{\Delta(1),N(2)...N(N)}(m_{\nu_1}...m_{\nu_N})$$

$$(\frac{P_1^2}{2M_1}+\frac{P_2^2}{2M_2}+\Delta M_1 - E_{\beta_1,...\beta_N}+E_{\alpha_3,...\alpha_N})_{m_{\nu_1}...m_{\nu_N};m_{\nu_1'}...m_{\nu_N'}}\phi_{\Delta(1),N(2)...N(N)}(m'_{\nu1}...m'_{\nu_N})]^{-1}.$$

The resulting wave function of the ΔN system in the closed shell nuclei with the ls- coupling is given in an explicit form in the work [11](s. Appendix 1.)

Formulae given in work [11] can be used for the closed shell nuclei $^4He,^{16}O$. However, the nucleus ^{12}C has not the closed 1p shell in simple shell- model description with the ls- coupling. To build a wave function of the isobar configuration with one delta in the case of the nucleus ^{12}C, we used the shell model with the jj- coupling, in which the nucleus ^{12}C has closed $s_{1/2}$ and $p_{3/2}$ shells. As a radial parts of the one- particle wave functions for the case the jj- coupling and the case the ls- coupling coincide for oscillator potential, the relationship between these wave functions is defined by the angular parts of the wave functions. Result, we have

$$\phi_{n_i'l_i'j_i'm_{j_i'}t_i'm_{t_i}'}(\vec{r}) = \sum_{m_{l_i'}m_{s_i'}} C^{j_i'm_i'}_{l_i'm_{l_i'}s_i'm_{S_i'}}\phi_{n_i'l_i'm_{l_i'}s_i'm_{S_i}'t_i'm_{t_i'}}(\vec{r}).$$

It allows to obtain a correlation between the wave functions for the ΔN system in the case of the shell model with the jj-and the ls-couplings

$$\Psi^{\Delta N}_{\alpha_1'\alpha_2'}(1,2) = \sum_{m_{l_1'}m_{l_2'}}\sum_{m_{S_1'}m_{S_2'}} C^{j_1'm_1'}_{l_1'm_{l_1'}s_1'm_{S_1'}}C^{j_2'm_2'}_{l_2'm_{l_2'}s_2'm_{S_2'}}\Psi^{\Delta N}_{\beta_1'\beta_2'}(1,2).$$

Here, $\alpha_i' = n_i'l_i'j_i'm_{j_i'}t_i'm_{t_i'}$, $\beta_i' = n_i'l_i'm_{l_i'}s_i'm_{s_i'}t_i'm_{t_i'}$. The wave functions of the ΔN system for shell model with the jj- coupling is given in an explicit form in Appendix 1.

Δ- isobar momentum distribution in nuclei

By definition, the Δ- isobar momentum distribution can be written through the wave function of the ΔN system as follows

$$\Delta_{\delta_1'\delta_2'}(\vec{k}) = \int d\hat{\vec{k}} d\vec{p}_1 d\vec{p}_2 \Psi^{+\Delta N}_{\delta_1'\delta_2'}(\vec{p}_1,\vec{p}_2)\delta(\vec{k}-\vec{p}_1)\Psi^{\Delta N}_{\delta_1'\delta_2'}(\vec{p}_1,\vec{p}_2).$$

The indices δ_i' (i = 1) are identified with the indices β_i' in the case of the ls-coupling and the indices α_i' in the case of the jj-coupling. Here, $\Psi^{\Delta N}_{\delta_1'\delta_2'}(\vec{p}_1,\vec{p}_2)$ is the Fourier transform of the wave function of the ΔN system

$$\Psi^{\Delta N}_{\delta_1'\delta_2'}(\vec{p}_1,\vec{p}_2) = \frac{1}{(2\pi)^3}\int d\vec{r}_1 d\vec{r}_2 \Psi^{\Delta N}_{\delta_1'\delta_2'}(\vec{r}_1,\vec{r}_2)e^{-i\vec{p}_1\vec{r}_1}e^{-i\vec{p}_2\vec{r}_2}.$$

Using the wave functions mentioned above, we have

$$\Psi^{\Delta N}_{\delta_1'\delta_2'}(\vec{p}_1,\vec{p}_2) = \sum_{\alpha}\Psi^{\Delta N}_{\alpha\delta_1'\delta_2'}(\vec{p})\Phi_\alpha(\vec{P}),$$

where $\alpha = N'L'M_{L'}$. Here, $\Psi^{\Delta N}_{\alpha\delta_1'\delta_2'}(\vec{p})$ is the Fourier transform of the function of relative motion and $\Phi_\alpha(\vec{P})$ is the Fourier transform of the wave function of c.m. system

$$\Psi^{\Delta N}_{N'L'M_{L'}\delta_1'\delta_2'}(\vec{p}) = \sum_{\gamma',lsJM_JTM_T}\Psi^{\Delta N;JM_JTM_Tls}_{N'L'M_{L'}\delta_1'\delta_2';\gamma'}(p) < \hat{\vec{p}} \mid (ls)JM_JTM_T >,$$

where

$$\Psi^{\Delta N;JM_JTM_Tls}_{N'L'M_{L'}\delta_1'\delta_2';\gamma'}(p) = F^{JM_JL'M_{L'}}_{\delta_1''\delta_2''s'l'\Lambda'}C^{Tm_T}_{t_1'm_{t_1'}t_2'm_{t_2'}}a^{n_1'l_1'n_2'l_2'\Lambda'}_{n'l'N'L'}(1-(-)^{l'+s'+T})W^{\Delta NJTM_JM_T}_{ls,n'l's'}(p),$$

$$W^{\Delta NJTM_JM_T}_{ls,n'l's'}(p) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int drr^2 4\pi(-i)^l j_l(pr)W^{\Delta NJTM_JM_T}_{ls,n'l's'}(r),$$

$$\Phi_{N'L'M_{L'}}(\vec{P}) = R_{N'L'}(P) < \hat{\vec{P}} \mid L'M_{L'} >,$$

and

$$R_{N'L'}(P) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int drr^2 4\pi(-i)^l j_l(Pr)R_{N'L'}(r).$$

The indices δ_i'' (i = 1) are identified with the indices $\beta_i'' = l_i'm_{l_i'}s_i'm_{s_i'}$ in the case of the ls-coupling and the indices $\alpha_i''= l_i'j_i'm_{j_i'}s_i'$ in the case of the jj-coupling. The momentum distribution integrated over angular variables is considered. Then, we can to write the momentum distribution of the Δ- isobar as follows

$$\Delta_{\delta_1'\delta_2'}(k) = \sum_{N'L'M_{L'};\tilde{N}'\tilde{L}'\tilde{M}_{L'}}\ \sum_{\gamma',lsJTM_JM_T;\tilde{\gamma}',\tilde{l}\tilde{s}\tilde{J}\tilde{T}\tilde{M}_J\tilde{M}_T}\int p^2dpP^2dP\Psi^{+\Delta N;JM_JTM_Tls}_{N'L'M_{L'}\delta_1'\delta_2';\gamma'}(p)$$

$$\Psi^{\Delta N;JM_JTM_Tls}_{N'L'M_{L'}\delta_1'\delta_2';\gamma'}(p)R_{N'L'M_{L'}}(P)R_{\tilde{N}'\tilde{L}'\tilde{M}_{L'}}(P)$$

$$A^{L'M_{L'};(ls)JM_JTM_T}_{\tilde{L}'\tilde{M}';(\tilde{l}\tilde{s})\tilde{J}\tilde{M}_J\tilde{T}\tilde{M}_T}(k,p,P),$$

where $A^{L'M_{L'};(ls)JM_JTM_T}_{\tilde{L}'\tilde{M}';(\tilde{l}\tilde{s})\tilde{J}\tilde{M}_J\tilde{T}\tilde{M}_T}(k,p,P)$ is the integral over the angular variables

$$A^{L'M_{L'};(ls)JM_JTM_T}_{\tilde{L}'\tilde{M}';(\tilde{l}\tilde{S})\tilde{J}\tilde{M}_J\tilde{T}\tilde{M}_T}(k,p,P) = \int d\hat{\vec{k}} d\hat{\vec{p}} d\hat{\vec{P}} < JM_J(ls)TM_T|\hat{\vec{p}} >< L'M_{L'}|\hat{\vec{P}} >$$

$$\delta(\vec{k} - \vec{p} - \frac{M_1}{M}\vec{P}) < \hat{\vec{p}} \mid \tilde{L}'\hat{M}_{L'} >< \hat{\vec{p}}|\hat{l}\tilde{s})\tilde{J}\tilde{M}_J\tilde{T}\tilde{M}_T > .$$

It is possible to show that

$$A^{L'M_{L'};(ls)JM_JTM_T}_{\tilde{L}'\tilde{M}';(\tilde{l}\tilde{s})\tilde{J}\tilde{M}_J\tilde{T}\tilde{M}_T}(k,p,P) = (-1)^{s+J+\tilde{l}+\tilde{J}}\frac{2}{\pi}\delta_{T\tilde{T}}\delta_{M_T\tilde{M}_T}\delta_{\tilde{s}s}\hat{l}\hat{L}'\hat{J}\hat{\tilde{J}}\sum_{\lambda m_\lambda}(-1)^{\lambda-m_\lambda}\hat{\lambda}^2$$

$$C^{\tilde{L}'0}_{L'0\lambda0}C^{\tilde{l}0}_{l0\lambda0}\left\{\begin{array}{c}\lambda\tilde{J}J\\ Sl\tilde{l}\end{array}\right\}$$

$$(-1)^{\tilde{L}'-M_{\tilde{L}'}}(-1)^{\tilde{J}-M_{\tilde{J}}}\left(\begin{array}{c}\lambda\tilde{J}J\\ -m_\lambda - M_{\tilde{J}}M_J\end{array}\right)\left(\begin{array}{c}L'\lambda\tilde{L}'\\ M_{L'}m_\lambda - M_{\tilde{L}'}\end{array}\right)$$

$$\int q^2 dq j_0(kq) j_\lambda(pq) j_\lambda(\frac{M_1}{M}Pq).$$

Below, the momentum distribution is considered as sum over all possible magnetic quantum numbers of the closed shells of the initial nucleons

$$\Delta_{\overline{\delta}'_1\overline{\delta}'_2}(k) = \sum \Delta_{\delta'_1\delta'_2}(k),$$

where $\overline{\delta}'_i = \overline{\beta}'_i = n'_i l'_i s'_i t'_i$ for the case of the ls- coupling and $\overline{\delta}'_i = \overline{\alpha}'_i = n'_i l'_i j'_i t'_i$ for the case of the jj- coupling. The sum over the quantum numbers $m_{l'_1}m_{l'_2}m_{s'_1}m_{s'_2}m_{t'_1}m_{t'_1}$ for the case of the ls- coupling and $m_{j'_1}m_{j'_2}m_{t'_1}m_{t'_1}$ for the case of the jj- coupling is supposed. After using of the graphic methods of the summation over the projections of the angular momenta [13], we can write the expression for the momentum distribution for the cases of the ls- and the jj- couplings in the forms, which are given in Appendix 2.

The momentum distributions of Δ - isobar for the nuclei ^4He and ^{16}O

For the simplest nucleus with the closed shell ^4He $n'_1 = n'_2 = l'_1 = L'_2 = 0$. Therefore, we have $2n' + 2N' + l' + L' = 0$. As all terms are more than zero, $n' = N' = l' = L' = 0$. From here, the following rules of the vector addition for the angular momenta must fulfill $(\tilde{J}J\lambda), (\lambda00), (J0s'), (s'\tilde{J}\tilde{l}')$ (the rule of triangle). Similarly, it have place $(00\lambda), (\lambda00), (00\Lambda'), \Lambda'00)$. At last, we have $(\tilde{J}J\lambda), (\lambda l\tilde{l}), Jls), (s\tilde{J}\tilde{l})$ from the third 6j- symbol. The transition potential gives the limitations l = 0, $(Jls), (s2s'), l2s'), (0Js'), (02l), (s_1 = 3/2 s_2 = 1/2 s)$. From here, we have

$$\lambda = 0, \Lambda' = 0, \tilde{J} = J, l = \tilde{l}, J = s', l = 0, 2, s(s', l),$$

where $s(s', 0) = J = s'$, $s(0, 2) = 2$ because of $(J = 0sl)$ and $s(1, 2) = 1, 2$ because of $(J = 1sl)$. The latter follows from the 9j- symbol properties.

As result, we shall receive

$$\Delta_{\overline{\beta}'_1\overline{\beta}'_2} = \sum_{s'=0}^{1}\sum_{J=s'}\sum_{T=0}^{1}\sum_{l=0,2}\sum_{s(s',l)} M^{slJslJT}_{00s';00s'}(0)$$

$$\int q^2dqj_0(kq)\int P^2dPj_0(\frac{M_\Delta}{M}Pq)R_{00}(P)^2\int p^2dpj_0(pq)|\,W^{\Delta NJT}_{ls,00s'}(p)\,|^2,$$

where

$$M^{slJslJT}_{00s';00s'}(0)=(-1)^{s+l}(1-(-1)^{s'+T})^2\frac{2}{\pi}\hat{l}\hat{J}^4\hat{T}^2N^{slJlJ}_{s'000}(0)$$

and

$$N^{slJlJ}_{s'000}(0)=\{JJ0\}\{000\}\left\{\begin{array}{c}JJ0\\00s'\end{array}\right\}\left\{\begin{array}{c}000\\000\end{array}\right\}\left\{\begin{array}{c}JJ0\\lls\end{array}\right\}.$$

Using properties of the 3j- and 6j- symbols, summing over l and s', T, we shall receive

$$\Delta_{\overline{\beta}'_1\overline{\beta}'_2}=\frac{24}{\pi}\int q^2dqj_0(kq)\int P^2dPj_0(\frac{M_\Delta}{M}Pq)R_{00}(P)^2\int p^2dpj_0(pq)$$

$$({}^1S_0+{}^5D_0-{}^3S_1-{}^3D_1-{}^5D_1),$$

where

$${}^1S_0=|\,W^{\Delta N01}_{00,000}(p)\,|^2,{}^5D_0=|\,W^{\Delta N01}_{22,000}(p)\,|^2$$

$${}^3S_1=|\,W^{\Delta N10}_{01,001}(p)\,|^2,{}^3D_1=|\,W^{\Delta N10}_{21,001}(p)\,|^2,{}^5D_1=|\,W^{\Delta N10}_{22,001}(p)\,|^2).$$

It is possible to perform the q' integration of the last expression. Besides, it is possible to show, that the matrix element of the transition potential is not zero only for the value of ${}^5D_0$. The values ${}^3S_1,{}^3D_1,{}^5D_1$ include the matrix element of the transition potential with T = 0, and so, they are equal to zero because of (03/21/2). The value ${}^1S_0$ is equal to zero because of the rule for the vector addition (03/21/2) for the spin angular momentum . Therefore, the momentum distribution of the Δ- isobar in the nucleus ${}^4He$ is

$$\Delta_{\overline{\beta}'_1\overline{\beta}'_2}=\frac{24}{(2\pi)^4}\frac{1}{\alpha}\sqrt{\frac{\pi}{2\nu}}\int pdp\frac{1}{k}(exp(-\frac{(k-p)^2}{2\alpha^2\nu})exp(-\frac{(k+p)^2}{2\alpha^2\nu}))|\,W^{\widetilde{\Delta N01}}_{22,000}(p)\,|^2.$$

Here,

$$W^{\widetilde{\Delta N01}}_{22,000}(p)=16\mu_{\Delta N}B_{\Delta N}\{\int_0^\infty r^2drj_2(pr)k_2(B_{\Delta N}r)\int_0^r r'^2dr'i_2(B_{\Delta N}r'V^{01}_{22,00}(r')R_{00}(r')+$$

$$\int_0^\infty r^2drj_2(pr)i_2(B_{\Delta N}r)\int_r^\infty r'^2dr'k_2(B_{\Delta N}r')V^{01}_{22,01}(r')R_{00}(r')\}.$$

The numerical calculations of the momentum distribution of the Δ- isobar for the nucleus ${}^4He$ are shown on Fig. 1 (curve a). The probability for finding one Δ- isobar per nucleon for the nucleus ${}^4He$ is given in Table 2.

For the nucleus ${}^{16}O$ the following sets of the main quantum numbers and the angular momentums are possible. Because of a centrifugal barrier two nucleons have the maximal probability to turn into a pair ΔN only in the state with $l'=0$. Therefore, the sum over the initial quantum numbers are simplified greatly. The possible initial quantum number are given in Table 1.

$n'_1=0$	$l'_1=0$	$n'_2=0$	$l'_2=0$	$n'=0$	$N'=0$	$l'=0$	$L'=0$	$\Lambda'=0$
$n'_1=0$	$l'_1=1$	$n'_2=0$	$l'_2=0$	$n'=0$	$N'=0$	$l'=0$	$L'=1$	$\Lambda'=1$
$n'_1=0$	$l'_1=0$	$n'_2=0$	$l'_2=1$	$n'=0$	$N'=0$	$l'=0$	$L'=1$	$\Lambda'=1$
$n'_1=0$	$l'_1=1$	$n'_2=0$	$l'_2=1$	$n'=0$	$N'=0$	$l'=0$	$L'=2$	$\Lambda'=2$
				$n'=1$	$N'=0$	$l'=0$	$L'=0$	$\Lambda'=0$
				$n'=0$	$N'=1$	$l'=0$	$L'=0$	$\Lambda'=0$

Table 1. The initial quantum numbers for the nucleus ${}^{16}O$.

If $l' = 0$, the rule of the vector addition of the angular momentum $(\lambda l' \tilde{l}')$ gives λ=0. Then, we have

$$M^{slJ\tilde{s}\tilde{l}\tilde{J}T}_{N'L'\gamma';\tilde{N}'\tilde{L}'\tilde{\gamma}'}(0) = (-1)^{s+\tilde{l}+\tilde{L}'+J-s'-\Lambda of'}(1-(-1)^{s'+T})(1-(-1)^{\tilde{s}'+T})\hat{T}^2$$

$$\frac{2}{\pi}\delta_{s'\tilde{s}'}\delta_{\Lambda'\tilde{\Lambda}'}\delta_{s\tilde{s}}\hat{l}\hat{J}^2\hat{\tilde{J}}^2\hat{\Lambda'}^2\hat{L}'\hat{\lambda}^2 C^{\tilde{L}'0}_{L'0\lambda 0}C^{00}_{0000}$$

$$a^{n_1'l_1'n_2'L_2'\Lambda'}_{n'l'N'L'}\, a^{n_1'l_1'n_2'l_2'\tilde{\Lambda}'}_{\tilde{n}'\tilde{l}'\tilde{N}'\tilde{L}'}\, N^{slJ\tilde{l}\tilde{J}}_{s'0L'\Lambda'0\tilde{\Lambda}'}(0),$$

where

$$N^{slJ\tilde{l}\tilde{J}}_{s'0L'\Lambda'0\tilde{L}'\tilde{\Lambda}'}(0) = \{\tilde{J}J0\}\{\tilde{L}'L'0\left\{ \begin{array}{c} \tilde{J}J0 \\ 00s' \end{array} \right\}\left\{ \begin{array}{c} \tilde{L}'L'0 \\ 00\Lambda' \end{array} \right\}\left\{ \begin{array}{c} \tilde{J}J0 \\ l\tilde{l}s \end{array} \right\}$$

and $\gamma' = s^{'}\Lambda^{'}n^{'}l^{'}$. Using properties the 3j-and 6j- symbols, we receive

$$N^{slJ\tilde{l}\tilde{J}}_{s'0L'\Lambda'0\tilde{L}'\tilde{\Lambda}'}(0) = (-1)^{s+\tilde{l}+\tilde{L}'+2\tilde{J}+s'+\Lambda'}\delta_{J\tilde{J}}\delta_{L'\tilde{L}'}\delta_{l\tilde{l}}\frac{1}{\hat{\tilde{J}}^2\hat{l}\hat{\tilde{L}}'}.$$

Hence (s. Appendix 2),

$$M^{slJ\tilde{s}\tilde{l}\tilde{J}T}_{N'L'\gamma';\tilde{N}'\tilde{L}'\tilde{\gamma}'}(0) = (-1)^{J}(1-(-1)^{s'+T})^2$$

$$\frac{2}{\pi}\delta_{s'\tilde{s}'}\delta_{\Lambda'\tilde{\Lambda}'}\delta_{L'\tilde{L}'}\delta_{s\tilde{s}}\delta_{l\tilde{L}}\delta_{J\tilde{J}}\hat{T}^2\hat{J}^2\hat{\Lambda'}^2$$

$$a^{n_1'l_1'n_2'L_2'\Lambda'}_{n'l'N'L'}\, a^{n_1'l_1'n_2'l_2'\tilde{\Lambda}'}_{\tilde{n}'\tilde{l}'\tilde{N}'\tilde{L}'},$$

where $\gamma' = s^{'}\Lambda^{'}n^{'}(l^{'} = 0)$. It gives the following expression for the momentum distribution of the Δ- isobar in the nucleus ^{16}O

$$\Delta_{\overline{\beta}_1'\overline{\beta}_2'} = \sum_{N'\tilde{N}'L'\tilde{L}'n'\tilde{n}'s'\Lambda'\tilde{\Lambda}'}\sum_{lsJT}(-1)^{-J+L'-\tilde{L}'}(1-(-1)^{s'+T})^2\frac{2}{\pi}\hat{T}^2\hat{J}^2\hat{\Lambda'}^2 a^{n_1'l_1'n_2'L_2'\Lambda'}_{n'l'N'L'}\, a^{n_1'l_1'n_2'L_2'\tilde{\Lambda}'}_{\tilde{n}'\tilde{l}'\tilde{N}'\tilde{L}'}\delta_{L'\tilde{L}'}\delta_{\Lambda'\tilde{\Lambda}'}$$

$$\int q^2 dq j_0(kq)\int P^2 dP j_0(\frac{M_\Delta}{M}Pq)R_{N'L'}R_{\tilde{N}'\tilde{L}'}(P)\int p^2 dp j_0(pq)W^{\Delta NJT}_{ls,n'l's'}(p)W^{\Delta NJT}_{ls,\tilde{n}'l's'}(p),$$

where $l' = \tilde{l}' = 0$. The sum over the initial quantum numbers is carried out according to the Table 1 at the fixed l_i'. Depending on a shell, where the two initial nucleons turn into a pair ΔN, there are three different combinations of the values l_1', l_2'. The sum momentum distribution of the Δ- isobar is

$$\Delta(k) = \Delta_{01\frac{1}{2}\frac{1}{2};01\frac{1}{2}\frac{1}{2}}(k) + \Delta_{01\frac{1}{2}\frac{1}{2};00\frac{1}{2}\frac{1}{2}}(k) + \Delta_{00\frac{1}{2}\frac{1}{2};01\frac{1}{2}\frac{1}{2}}(k) + \Delta_{00\frac{1}{2}\frac{1}{2};00\frac{1}{2}\frac{1}{2}}(k).$$

The sum over s', l, s, J, T is carried out as well as in the case of the nucleus ^4He. The identical the limitations follow because of the same transition potential. In this case $l = 0, 2, J = s'$, $s(s', 0) = s'$, $s(0, 2) = 2$ because of $(J = 0sl)$, $s(1, 2) = 1, 2$ because of $(J = 1sl)$. As in the previous case, because of the rules of the vector addition (03/21/2)for T and (03/21/2)for the spin angular momentum, the transitions with T = 0 and the transitions with $s' = 0, l = 0, s = 0, J = 0, T = 1$ are absent. As result, the momentum distribution of the Δ- isobar in the nucleus ^{16} O is

$$\Delta(k) = \frac{24}{\pi}\int q^2 dq j_0(kq)\int P^2 dP j_0(\frac{M_\Delta}{M}Pq)\int p^2 dp j_0(pq)[$$

$$(R_{00}(P)^2) \mid W^{\Delta N01}_{22,000}(p) \mid^2 +$$

$$+3R_{01}(P)^2 \mid W^{\Delta N01}_{22,000}(p) \mid^2 +$$

$$(\frac{5}{2}R_{02}(P)^2+\frac{1}{2}R_{10(P)^2})\mid W^{\Delta N01}_{22,000}(p) \mid^2 - R_{10}(P)R_{00}(P)W^{\Delta N01}_{22,000}(p)W^{\Delta N01}_{22,100}(p)+\frac{1}{2}R_{00}(P)^2 \mid W^{\Delta N01}_{22,100}(p) \mid^2].$$

The Fourier transform of the radial parts are determined by formula

$$R_{nl}(p) = \frac{4\pi}{(2\pi)^3/2}(-i)^l \int r^2 dr j_l(pr) R_{nl}(r).$$

After an analytical calculation of the available integrals, we shall receive the following expression for the momentum distribution of the Δ- isobar in the nucleus ^{16}O

$$\Delta(k) = \frac{24}{(2\pi)^4}\frac{1}{\alpha}\sqrt{\frac{\pi}{2\nu}}\frac{1}{k}\int pdp$$

$$(exp(-\frac{(k-p)^2}{2\alpha^2\nu})f_1(k-p) - exp(-\frac{(k+p)^2}{2\alpha^2\nu})f_1(k+p)\mid W^{\widetilde{\Delta N01}}_{22,000}(p) \mid^2$$

$$+\sqrt{6}(exp(-\frac{(k-p)^2}{2\alpha^2\nu})f_2(k-p) - exp(-\frac{(k+p)^2}{2\alpha^2\nu}f_2(k+p)))W^{\widetilde{\Delta N01}}_{22,000}(p)W^{\widetilde{\Delta N01}}_{22,100}(p)$$

$$+(exp(-\frac{(k-p)^2}{2\alpha^2\nu})f_3(k-p) - exp(-\frac{(k+p)^2}{2\alpha^2\nu}f_3(k+p)) \mid W^{\widetilde{\Delta N01}}_{22,100}(p) \mid^2),$$

where

$$f_1(x) = 6 + \frac{1}{4}\frac{x^2}{\alpha^2\nu} + \frac{1}{4}\frac{x^4}{\alpha^4\nu^2}$$

$$f_2(x) = \frac{1}{6}(1 - \frac{x^2}{\alpha^2\nu})$$

$$f_3(x) = \frac{1}{2}.$$

Here, $\alpha = \frac{M_\Delta}{M_\Delta + M_N}$

Using the formula written above, we have obtained the momentum distribution of the Δ- isobar for the nucleus ^{16}O. Results are shown on Fig. 1 (curve b). The probability for finding of the Δ- isobar per nucleon in the nucleus ^{16}O is given in Table 2.

Momentum distribution of Δ - isobar for the nucleus ^{12}C

Let us consider the momentum distribution of the Δ - isobar in the ground state of the nucleus ^{12} C. This nucleus in the ground state have two closed $s_{1/2}$ and $l_{3/2}$ shells. The sum over the quantum number for the momentum distribution of the Δ - isobar for the nucleus ^{12}C (s. the formulae of Appendix 2) is defined by the rules of the vector addition for the angular momentums. Below, we shall consider the case $l' = 0$. By definition, we have $s'_1 = \frac{1}{2}, s'_2 = \frac{1}{2}, s_1 = \frac{3}{2}, s_2 = \frac{1}{2}$. From the form of the transition potential it follows l = 0 or l = 2. For these two cases $s' = 0, 1$. From the 6j- symbols in the isotopic space it follows T = 1. Then, because of the factor $(1-(-1)^{l'+s'+T})$, the selection rule for spin follows $s' = \tilde{s}' = 0$. It is very strong limitation. We have $J = 0$ and $\lambda = 0$. In addition, $l = \tilde{l}$ and $s = l$. It follows $x = L' = \tilde{L}' = \Lambda' = \tilde{\Lambda}'$ also. Because of the first part of the transition potential the value l = 0. However, because of (03/21/2) the contribution of this

item disappears. Therefore, only $l = s = 2$ remains. Thus, the expression for the momentum distribution becomes considerably simpler

$$\Delta_{\overline{\alpha}'_1\overline{\alpha}'_2}(k) = \sum_{N'n'\tilde{N}\tilde{n}'} \sum_{L'\Lambda'} M^{\lambda=0l=s=2J=0\tilde{l}=2\tilde{J}=0T=1}_{N'L'n'l'=0s'=0\Lambda';\tilde{N}'L'\tilde{n}'l'=0s'=0\Lambda'}(\overline{\alpha}'_1\overline{\alpha}'_2) \int q^2 dq j_0(kq)$$

$$\int P^2 dP j_\lambda(\frac{M_\Delta}{M}Pq) R_{N'L'}(P) R_{\tilde{N}'L'}(P) \int p^2 dp j_\lambda(pq) W^{\Delta NJ=0T=1}_{l=2s=2n'l'=0s'=0}(p) W^{\Delta N\tilde{J}=0T=1}_{\tilde{l}=2s=2,\tilde{n}'\tilde{l}'=2\tilde{s}'=2}(p).$$

Let us designate the momentum distributions, when both holes are on the $p_{\frac{3}{2}}$ - shells, as follows

$$\Delta_{p_{\frac{3}{2}}p_{\frac{3}{2}}} = \Delta_{\overline{\alpha}'_1\overline{\alpha}'_2}(k),$$

where we have $\overline{\alpha}'_1 \equiv (n'_1 = 0 l'_1 = 1 j'_1 = 3/2 t'_1 = 1/2)$ $\overline{\alpha}'_2 \equiv (n'_2 = 0 l'_2 = 1 j'_2 = 3/2 t'_2 = 1/2)$. Similarly, we shall designate the momentum distribution, when one hole is on the $s_{\frac{1}{2}}$, and other hole is on the $p_{\frac{3}{2}}$ - shell, as follows

$$\Delta_{p_{\frac{1}{2}}p_{\frac{3}{2}}} = \Delta_{\overline{\alpha}'_1\overline{\alpha}'_2}(k),$$

where $\overline{\alpha}'_1 \equiv (n'_1 = 0 l'_1 = 0 j'_1 = 1/2 t'_1 = 1/2)$ $\overline{\alpha}'_2 \equiv (n'_2 = 0 l'_2 = 1 j'_2 = 3/2 t'_2 = 1/2)$. At last, we shall designate the momentum distribution, when both holes are on the $s_{\frac{1}{2}}$ - shells, as follows

$$\Delta_{s_{\frac{1}{2}}s_{\frac{1}{2}}} = \Delta_{\overline{\alpha}'_1\overline{\alpha}'_2}(k),$$

where $\overline{\alpha}'_1 \equiv (n'_1 = 0 l'_1 = 0 j'_1 = 1/2 t'_1 = 1/2)$ $\overline{\alpha}'_2 \equiv (n'_2 = 0 l'_2 = 0 j'_2 = 1/2 t'_2 = 1/2)$. Then, the momentum distribution of the Δ- isobars for the nucleus ^{12} C is a sum of the momentum distributions for three various cases

$$\Delta(k) = \Delta_{p_{\frac{3}{2}}p_{\frac{3}{2}}} + \Delta_{p_{\frac{3}{2}}p_{\frac{2}{3}}} + \Delta_{p_{\frac{2}{3}}p_{\frac{3}{2}}} + \Delta_{s_{\frac{1}{2}}s_{\frac{1}{2}}}.$$

Using the results of the previous sections for a calculation of the integrals and the values $W^{\Delta N01}_{22,000}(p)$ and $W^{\Delta N01}_{22,100}(p)$ with appropriate replacement of parameters of oscillator model for ^{16}O on parameters for ^{12}C, we have

$$\Delta(k) = \frac{16}{(2\pi)^4}\frac{1}{2\alpha}\sqrt{\frac{\pi}{2\nu}}\frac{1}{k}\int pdp\{[exp(-\frac{(k-p)^2}{2\alpha^2\nu})f_1(k-p) - exp(-\frac{(k+p)^2}{2\alpha^2\nu})f_1(k+p)]W^{\Delta\tilde{N01}}_{22,000}(p)^2 +$$

$$[exp(-\frac{(k-p)^2}{2\alpha^2\nu})f_2(k-p) - exp(-\frac{(k+p)^2}{2\alpha^2\nu})f_2(k+p)]W^{\Delta\tilde{N}01}_{22,000}(p)W^{\Delta\tilde{N}01}_{22,100}(p) +$$

$$[exp(-\frac{(k-p)^2}{2\alpha^2\nu})f_3(k-p) - exp(-\frac{(k+p)^2}{2\alpha^2\nu})f_3(k+p)]W^{\Delta\tilde{N01}}_{22,100}(p)^2.$$

Here,

$$f_1(x) = \frac{11}{2} + 2(2 + \frac{x^2}{\alpha^2\nu}) + \frac{1}{3}(2 - \frac{x^2}{\alpha^2\nu})^2$$

$$f_2(x) = \frac{2}{\sqrt{6}}(1 - \frac{x^2}{\alpha^2\nu})$$

$$f_3(x) = 1.$$

The numerical calculations of the momentum distribution of the Δ- isobar for the nucleus ^{12}C are shown on Fig. 1 (curve c). The probability for finding of the Δ- isobar per nucleon in the nucleus ^{12}C is given in Table 2.

Nucleus	$P_{1\Delta}$	C	N	P_{total}	P_m
^4He	2.73	0.8907	0.1227	0.1094	0.1081
^{12}C	2.19	0.8582	0.3574	0.2633	0.2604
^{16}O	2.17	0.8074	0.5338	0.3401	0.3448

Table 2. The probability for finding of the Δ- isobar in the nucleus per nucleon $P_{1\Delta}$ is given in %; C is the constant of the normalization; N is the norm of the wave function of the Δ- isobar; P_t is the full probability for finding of the Δ- isobar in the nucleus. P_m is the result of the k integration of the value $k^2\Delta(k)$.

Conclusions

The wave function of the Δ- isobar configuration in closed shell nuclei was obtained in [11] in the harmonic oscillator model with the ls- coupling. The transition potential with π- and ρ- exchange was used. We use this result for building of the wave function of the Δ- isobar configuration in the case of the harmonic oscillator model with the jj- coupling. The probability for finding of the Δ- isobar per nucleon in the nucleus and the momentum distribution of the Δ- isobar were calculated for the closed shell nuclei ^4He and ^{16}O in the case of the harmonic oscillator model with the ls- coupling and the closed shell nucleus ^{12}C in the case of the harmonic oscillator model with the jj- coupling. This results can be used for a interpretation of the experimental data in reactions on nuclei in the new experiments connected with of the Δ knock-out from the nuclei target by π- meson and photon beams. In particular, the received results can be used for the interpretation of the experimental data in the reaction $^{12}C(\gamma,\pi^+p)$ in the framework of the assumption that formation of the π^+p pairs may be interpreted as a $\gamma\Delta^{++}\to\pi^+p$ process, which takes place on a Δ^{++} preexisting in the target nucleus.

Appendix 1.

a) The wave function of the Δ N system obtained in [11] in the harmonic oscillator shell-model with the ls- coupling can be presented as follows

$$\Psi^{\Delta N}_{\beta_1'\beta_2'}(\vec{r_1},\vec{r_2}) = \sum_{N'L'M_L'} \Psi^{\Delta N}_{\beta_1'\beta_2'N'L'M_L'}(\vec{r})\Phi_{N'L'M_L'}(\vec{R}).$$

The part of the wave function dependent on $\vec{r}$ can be written as follows

$$\Psi^{\Delta N}_{\beta_1'\beta_2'N'L'M_L'}(\vec{r}) = \sum_{lsJM_JTM_T} \Psi^{\Delta NlsJM_JTM_T}_{\beta_1'\beta_2'N'L'M_{L'}}(r)\langle\hat{\vec{r}}|\ (ls)JM_JTM_T\rangle.$$

Here,

$$\Psi^{\Delta NlsJM_JTM_T}_{\beta_1'\beta_2'N'L'M_{L'}}(r) = \sum_{\gamma'} \Psi^{\Delta N\, lsJM_JTM_T}_{\beta_1'\beta_2'N'L'M_{L'}\gamma'}(r),$$

$\beta_i' = n_i'l_i'm_{l_i}'s_i'm_{s_i}'t_i'm_{t_i}'$, $\gamma' = s'\Lambda'n'l'$ and

$$\Psi^{\Delta N\, lsJM_JTM_T}_{\beta_1'\beta_2'N'L'M_{L'}\gamma'}(r) = F^{JM_JL'M_{L'}}_{\beta_1''\beta_2''s'l'\Lambda'}\, C^{Tm_T}_{t_1'M_{t_1'}t_2'm_{t_2'}}\, a^{n_1'l_1'n_2'l_2'\Lambda'}_{n'l'N'L'}\, (1-(-)^{l'+s'+T})W^{\Delta NJTM_JM_T}_{ls,n'l's'}(r).$$

The value $W^{\Delta NJTM_JM_T}_{ls,n'l's'}(r)$ has the form

$$W^{\Delta NJTM_JM_T}_{ls,n'l's'}(r) = -\int_0^\infty dr^{'} r^{'2} G_l(r,r^{'};\mu_{12},b_{12}) V^{JTM_JM_T}_{ls,l's'}(r^{'}) R_{n'l'}(r^{'}),$$

$$V^{JTM_JM_T}_{ls,l's'}(r') = \int d\hat{\vec{r}'} \langle TM_TJM_J(ls) \mid \hat{\vec{r}'}\rangle V_{\Delta N,NN}(\vec{r}')\langle \hat{\vec{r}'} \mid TM_TJM_J(l's')\rangle.$$

By definition

$$F^{JM_JL^{'}M_{L'}}_{\beta''_1\beta''_2 s'l'\Lambda'} = \sum_{m_{s'}m_{l'}M_{\Lambda'}} C^{s^{'}m_{s'}}_{s'_1m_{s'_1}s'_2m_{s'_2}} C^{\Lambda^{'}M_{\Lambda'}}_{l'_1m_{l'_1}l'_2m_{l'_2}} C^{\Lambda^{'}M_{\Lambda'}}_{l'm_{l'}L'M_{L'}} C^{JM_J}_{l'm_{l'}s'm_{s'}}$$

and $\beta''_i = l'_im_{l'_i}s'_im_{s'_i}$. The projections of the angular momenta $M_J, M_{L'}, m_{l'_i}, m_{s'_i}$ are fixed. The two-particle propagator is equal

$$G_l(r,r';\mu_{\Delta N},B_{\Delta N}) =$$

$$= \frac{4}{\pi}\mu_{\Delta N}B_{\Delta N}k_l(B_{\Delta N}r)i_l(B_{\Delta N}r') \qquad r > r'$$

$$= \frac{4}{\pi}\mu_{\Delta N}B_{\Delta N}i_l(B_{\Delta N}r)k_l(B_{\Delta N}r') \qquad r < r'$$

$$\frac{1}{\mu_{\Delta N}} = \frac{1}{M_\Delta} + \frac{1}{M_N}$$

$$B_{\Delta N} = \sqrt{2\mu_{\Delta N}(\frac{K^2}{2M} + \Delta M_1 + \Delta M_2 - E_{(\alpha\beta)}}.$$

Here, $\Delta M_1 = M_\Delta - M$, $\Delta M_2 = M_N - M_M = 0$.

The transition potential was obtained in the OBE approximation (one-π and one-ρ exchanges) and have the form [11]

$$V_{\Delta N,NN}(\vec{r}) = \frac{m_\pi}{3}V^\pi_{\Delta N}(\vec{\tau_1}\cdot\vec{\tau_2})[(\vec{\sigma_1}\cdot\vec{\sigma_2})(v_0(m_\pi r) + 2\frac{m_\rho}{m_\pi}\frac{V^\rho_{\Delta N}}{V^\pi_{\Delta N}}v_0(m_\rho r)) +$$

$$S_{12}(v_2(m_\pi r) - \frac{m_\rho}{m_\pi}\frac{V^\rho_{\Delta N}}{V^\pi_{\Delta N}}v_2(m_\rho r))].$$

Here, the operator S_{12} is

$$S_{12} = \sqrt{24\pi}[\sigma_1^{[1]} \times \sigma_2^{[1]}]^{[2]} \times Y^{[2]}(\hat{r})]^{[0]}.$$

The matrix element of the transition potential

$$V^{JTM_JM_T;J'T'M_{J'}M_{T'}}_{ls,l's'}(r) = \int d\hat{\vec{r}'} < (l(s_1s_2)s)J(t_1t_2)T \mid \hat{\vec{r}'} > V_{\Delta N,NN}(\vec{r}')$$

$$< \hat{\vec{r}'} \mid (l^{'}(s^{'}_1s^{'}_2)s^{'})J^{'}M_{J'}(t^{'}_1t^{'}_2)T^{'}M_{T'} > .$$

is written as follows

$$V^{JTM_JM_T;J'T'M_{J'}M_{T'}}_{ls,l's'}(r) = \delta_{J'J}\delta_{T'T}\delta_{M_{J'}M_J}\delta_{M_{T'}M_T}V^{JT}_{ls,l's'}(r)$$

and

$$V^{JT}_{ls,l's'}(r)=\frac{m_\pi}{3}V^{\pi}_{\Delta N}[M^{JT}_{ls,l's'}(1)(v_0(m_\pi r)+2\frac{m_\rho}{m_\pi}\frac{V^{\rho}_{\Delta N}}{V^{\pi}_{\Delta N}}v_0(m_\rho r))+$$

$$M^{JT}_{ls,l's'}(2)(v_2(m_\pi r)-\frac{m_\rho}{m_\pi}\frac{V^{\rho}_{\Delta N}}{V^{\pi}_{\Delta N}}v_2(m_\rho r))].$$

Here,

$$M^{JT}_{ls,l's'}(1)=(-1)^{t_1+t_2^{'}+T}\left\{\begin{array}{c} Tt_2t_1 \\ 1t_1^{'}t_2^{'}\end{array}\right\}\times$$

$$<t_1\parallel\tau^{[1]}(1)\parallel t_1^{'}><t_2\parallel\tau^{[1]}(2)\parallel t_2^{'}><s_1\parallel\sigma^{[1]}(1)\parallel s_1^{'}><s_2\parallel\sigma^{[1]}(2)\parallel s_2^{'}>\times$$

$$(-1)^{s_1+s_2^{'}+s^{'}}\delta_{ll'}\left\{\begin{array}{c} s^{'}s_2s_1 \\ 1s_1^{'}s_2^{'}\end{array}\right\}$$

$$M^{JT}_{ls,l's'}(2)=(-1)^{t_1+t_2^{'}+T}\left\{\begin{array}{c} Tt_2t_1 \\ 1t_1^{'}t_2^{'}\end{array}\right\}\times$$

$$<t_1\parallel\tau^{[1]}(1)\parallel t_1^{'}><t_2\parallel\tau^{[1]}(2)\parallel t_2^{'}><s_1\parallel\sigma^{[1]}(1)\parallel s_1^{'}><s_2\parallel\sigma^{[1]}(2)\parallel s_2^{'}>\times$$

$$\sqrt{30}(-1)^{J+s}\hat{s^{'}}\hat{s}\hat{l}\hat{l^{'}}$$

$$\left(\begin{array}{c} l^{'}2l \\ 000\end{array}\right)\left\{\begin{array}{c} Jls \\ 2s^{'}l^{'}\end{array}\right\}\left\{\begin{array}{c} s_1s_1^{'}1 \\ s_2s_2^{'}1 \\ ss^{'}2\end{array}\right\}$$

and

$$v_l(r)=\frac{2}{\pi}(-1)^{l_1+l_2}\frac{\Lambda^4}{(\Lambda^2-m^2)^2}[k_l(mr)-(\frac{\Lambda}{m})^{l_1+l_2+1}[k_1(\Lambda r)+$$

$$\frac{\Lambda^2-m^2}{2\Lambda^2}(\Lambda r k_{l-1}(\Lambda r)-(l_1+l_2-l)k_l(\Lambda r)]].$$

Here,

$$V^{\pi}_{\Delta N}=\frac{f_{\Delta N\pi}f_{NN\pi}}{4\pi},$$

$$V^{\rho}_{\Delta N}=\frac{f_{\Delta N\rho}f_{NN\rho}}{4\pi},$$

Λ is the regularization parameter. For the constants we take the following values

$$\frac{V^{\rho}_{\Delta N}}{V^{\pi}_{\Delta N}}=3.9953[14].$$

and

$$V^{\pi}_{\Delta N}=0.17[15].$$

Therefore, the values written above are

$$V^{JTM_JM_T}_{ls,l's'}(r)=\delta_{JJ}\delta_{TT}\delta_{M_JM_J}\delta_{M_TM_T}V^{JT}_{ls,l's'}(r),$$

$$W^{JTM_JM_T}_{ls,l's'}(r)=\delta_{JJ}\delta_{TT}\delta_{M_JM_J}\delta_{M_TM_T}W^{JT}_{ls,l's'}(r).$$

Here,

$$W^{\Delta NJT}_{ls,n'l's'}(r)=-\int_0^{\infty}dr^{'}r^{'2}G_l(r,r^{'};\mu_{12},b_{12})V^{JT}_{ls,l's'}(r^{'})R_{n'l'}(r^{'}).$$

b) After transition from the coordinates $\vec{r_1}$, $\vec{r_2}$ to relative $\vec{r}$ and c.m. $\vec{R}$ coordinates the wave function of the ΔN system in the case the jj- coupling can be written as follows

$$\Psi^{\Delta N}_{\alpha_1^{'}\alpha_2^{'}}(\vec{r_1},\vec{r_2}) = \sum_{\alpha} \Psi^{\Delta N}_{\alpha_1^{\prime}\alpha_2^{\prime}\alpha}(\vec{r})\Phi_{\alpha}(\vec{R}),$$

where $\alpha = N^{'}L^{'}M_{L^{'}}$ and

$$\Psi^{\Delta N}_{\alpha\alpha_1^{\prime}\alpha_2^{\prime}}(\vec{r}) = \sum_{m_{l_1^{'}}m_{l_2^{'}}} \sum_{m_{s_1^{'}}m_{s_2^{'}}} C^{j_1^{'}m_1^{'}}_{l_1^{'}m_{l_1^{'}}s_1^{'}m_{S_1^{\prime}}} C^{j_2^{'}m_2^{'}}_{l_2^{\prime}m_{l_2^{'}}s_2^{'}m_{S_2^{\prime}}} \times \Psi^{\Delta N}_{\beta_1^{'}\beta_2^{\prime}\alpha}(\vec{r}).$$

Let us define

$$F^{JM_JL^{'}M_{L^{'}}}_{\alpha_1^{''}\alpha_2^{''}s^{\prime}l^{\prime}\Lambda^{\prime}} = \sum_{m_{l_1^{'}}m_{l_2^{'}}m_{s_1^{'}}m_{s_2^{'}}} C^{j_1^{'}m_{j_1^{'}}}_{l_1^{\prime}m_{l_1^{'}}s_1^{'}m_{s_1^{'}}} C^{j_2^{'}m_{j_2^{'}}}_{l_2^{\prime}m_{l_2^{'}}s_2^{'}m_{s_2^{'}}}$$

$$F^{JM_JL^{'}M_{L^{'}}}_{\beta_1^{''}\beta_2^{''}s^{\prime}l^{\prime}\Lambda^{\prime}},$$

where $\alpha_i^{''} = l_i^{'}j_i^{\prime}m_{j_i^{'}}s_i^{'}$. Then, the wave function of relative motion of the ΔN system can be written as follows

$$\Psi^{\Delta N}_{\alpha_1^{\prime}\alpha_2^{\prime}N^{\prime}L^{\prime}M_{L^{\prime}}}(\vec{r}) = \sum_{\gamma^{\prime},JM_JTM_Tls} \Psi^{\Delta NlsJM_JTM_T}_{\alpha_1^{\prime}\alpha_2^{\prime}N^{\prime}L^{\prime}M_L^{\prime};\gamma^{\prime}}(r) < \hat{\vec{r}} \,|\, (ls)JM_JTM_T >,$$

where $\gamma^{\prime} \equiv s^{'}\Lambda^{\prime}n^{'}l^{'}$. It its turn, the radial wave function is given by

$$\Psi^{\Delta NlsJM_JTM_T}_{\alpha_1^{\prime}\alpha_2^{\prime}N^{\prime}L^{\prime}M_L^{\prime};\gamma^{\prime}}(r) = F^{JM_JL^{'}M_{L^{'}}}_{\alpha_1^{''}\alpha_2^{''}s^{\prime}l^{\prime}\Lambda^{\prime}} C^{Tm_T}_{t_1^{\prime}m_{t_1^{'}}t_2^{\prime}m_{t_2^{'}}} a^{n_1^{'}l_1^{'}n_2^{'}l_2^{'}\Lambda^{'}}_{n^{'}l^{'}N^{'}L^{'}}(1-(-)^{l^{'}+S^{'}+T})W^{\Delta NJTM_JM_T}_{ls,n^{'}l^{'}s^{'}}(r).$$

It can be show that

$$F^{JM_JL^{'}M_{L^{'}}}_{\alpha_1^{''}\alpha_2^{''}s^{\prime}l^{\prime}\Lambda^{\prime}} = (-1)^{l_1^{'}+l_2^{'}+s_1^{'}-s_2^{'}+j_1^{'}+j_2^{'}+\Lambda^{'}+2l^{\prime}-2s^{\prime}-2J-2L^{\prime}}\hat{j_1^{'}}\hat{j_2^{'}}\hat{s}^{\prime}\hat{\Lambda}^{\prime 2}\hat{J}\sum\nolimits_{x\xi}(-1)^{x-\xi}(2x+1)$$

$$\begin{pmatrix} j_1^{'}xj_2^{'} \\ m_{j_1^{'}}\xi m_{j_2^{'}} \end{pmatrix} \begin{pmatrix} L^{'}xJ \\ M_{L^{'}}\xi M_J \end{pmatrix} \begin{Bmatrix} L^{'}xJ \\ s^{'}l^{'}\Lambda^{'} \end{Bmatrix} \begin{Bmatrix} j_1^{'}xj_2^{'} \\ l_1^{'}\Lambda^{'}l_2^{'} \\ s_1^{'}s^{'}s_2^{'} \end{Bmatrix}.$$

Here, {} are the 6j- 9j- symbols.

c) By definition, the norm of the wave function of the Δ N system is

$$N = \sum_{\delta_1^{\prime}\delta_2^{\prime}} \int \Psi^{+\Delta N}_{\delta_1^{\prime}\delta_2^{\prime}}(\vec{r}_1^{\prime}\vec{r}_2^{\prime})\Psi^{\Delta N}_{\delta_1^{\prime}\delta_2^{\prime}}(\vec{r}_1^{\prime}\vec{r}_2^{\prime})d\vec{r}_1^{\prime}d\vec{r}_2^{\prime}.$$

The indices $\delta_i^{\prime}$ (i = 1,2) are identified with the indices $\beta_i^{\prime}$ in the case of the ls-coupling and the indices $\alpha_i^{\prime}$ in the case the jj- coupling. Here, the sum over the spin and isospin variables is meant. After transition to relative coordinate and c. m. coordinate we have

$$N = \sum_{\delta_1^{\prime}\delta_2^{\prime}} \int d\vec{r} \sum_{N^{\prime}L^{\prime}M_{L^{\prime}}} \Psi^{\star\Delta N}_{\delta_1^{\prime}\delta_2^{\prime}N^{\prime}L^{\prime}M_{L^{\prime}}}(\vec{r})\Psi^{\Delta N}_{\delta_1^{\prime}\delta_2^{\prime}N^{\prime}L^{\prime}M_{L^{\prime}}}(\vec{r}).$$

For transformation of the last ratio we shall take the formulae mentioned above. Performing the angular variables integration, we have

$$N=\sum_{\delta_1'\delta_2'}\sum_{N'L'M_{L'}}\sum_{lsJM_JTM_T}\sum_{n'l's'\Lambda'}\sum_{\tilde{n}'\tilde{l}'\tilde{s}'\tilde{\Lambda}'}$$

$$F^{JM_JL'M_{L'}}_{\delta_1''\delta_2''s'l'\Lambda'}F^{JM_JL'M_{L'}}_{\delta_1''\delta_2''\tilde{s}'\tilde{l}'\tilde{\Lambda}'}C^{Tm_T}_{t_1'm_{t_1'}t_2'm_{t_2'}}C^{Tm_T}_{t_1'm_{t_1'}t_2'm_{t_2'}}a^{n_1'l_1'n_2'l_2'\Lambda'}_{n'l'N'L'}a^{n_1'l_1'n_2'l_2'\tilde{\Lambda}'}_{\tilde{n}'\tilde{l}'N'L'}$$

$$(1-(-)^{l'+s'+T})(1-(-)^{\tilde{l}'+\tilde{s}'+T})$$

$$\int W^{\star\Delta NJTM_JM_T}_{ls,n'l's'}(r)W^{\Delta NJTM_JM_T}_{ls,\tilde{n}'\tilde{l}'\tilde{s}'}(r)r^2dr.$$

Using the formulae for F mentioned above, we have the following expressions for the norm in case of the ls- coupling

$$N=\sum_{n_1'n_2'l_1'l_2'}\sum_{N'L'M_{L'}}\sum_{lsJT}\sum_{s'l'}\sum_{n'\tilde{n}'}\frac{\hat{\Lambda'}^2\hat{J}^2\hat{T}^2}{\hat{l'}^2}2(1-(-1)^{l'+s'+T})$$

$$a^{n_1'l_1'n_2'l_2'\Lambda'}_{n'l'N'L'}a^{n_1'l_1'n_2'l_2'\Lambda'}_{\tilde{n}'l'N'L'}\int W^{\star\Delta NJT}_{ls,n'l's'}(r)W^{\Delta NJT}_{ls,\tilde{n}'l's'}(r)r^2dr.$$

In the case of the jj- coupling the norm of the wave function of the isobar configuration can be written as follows

$$N=\sum_{n_1'l_1'j_1'n_2'l_2'j_2'}\int r^2dr\sum_{N'L'}\sum_{lsJT}\sum_{n'l's'\Lambda'}\sum_{\tilde{n}'\tilde{l}'\tilde{s}'\tilde{\Lambda}'}(-1)^{a+\tilde{a}}\hat{j}_1'^2\hat{j}_2'^2\hat{\tilde{\Lambda}}'^2\hat{\Lambda}'^2\hat{J}^2\sum_x\hat{x}^2\{j_1'j_2'x\}$$

$$\left\{\begin{matrix}L'xJ\\ s'l'\Lambda'\end{matrix}\right\}\left\{\begin{matrix}L'xJ\\ \tilde{s}'\tilde{l}'\tilde{\Lambda}'\end{matrix}\right\}\left\{\begin{matrix}j_1'xj_2'\\ l_1'\Lambda'l_2'\\ s_1's's_2'\end{matrix}\right\}\left\{\begin{matrix}j_1'xj_2'\\ l_1'\tilde{\Lambda}'l_2'\\ s_1'\tilde{s}'s_2'\end{matrix}\right\}.$$

$$a^{n_1'l_1'n_2'l_2'\Lambda'}_{n'l'N'L'}a^{n_1'l_1'n_2'l_2'\Lambda'}_{\tilde{n}'\tilde{l}'N'L'}$$

$$(1-(-)^{l'+s'+T})(1-(-)^{\tilde{l}'+\tilde{s}'+T})$$

$$W^{\Delta NJT}_{ls,n'l's'}(r)W^{\Delta NJT}_{ls,\tilde{n}'\tilde{l}'\tilde{s}'}(r).$$

Appendix 2.

With the help of the graphic method of summation over the projections of the angular momenta [13] it is possible to receive the following expression in the case of the ls- coupling

$$\sum_{m_{l_1}'m_{l_2}'m_{s_1}'m_{s_2}'}F^{JM_JL'M_{L'}}_{\beta_1''\beta_2''s'l'\Lambda'}F^{\tilde{J}\tilde{M}_J\tilde{L}'\tilde{M}_{L'}}_{\beta_1''\beta_2''\tilde{s}'\tilde{l}'\tilde{\Lambda}'}=$$

$$\sum_{xm_x}\beta(x)(-1)^{x-m_x}\hat{x}^2(-1)^{\tilde{L}'-M_{\tilde{L}'}}(-1)^{\tilde{J}-\tilde{M}_J}$$

$$\begin{pmatrix}\tilde{L}'xL'\\ -M_{\tilde{L}'}m_xM_{L'}\end{pmatrix}\begin{pmatrix}\tilde{J}xJ\\ -M_{\tilde{J}}-m_xM_J\end{pmatrix},$$

where

$$\beta(x)=\delta_{s'\tilde{s}'}\delta_{\Lambda'\tilde{\Lambda}'}(-1)^{-l'-\tilde{l}'-s'-\Lambda'-2L'+\tilde{L}'-J}\hat{\Lambda'}^2\hat{J}\hat{\tilde{J}}\left\{\begin{array}{c}\tilde{J}xJ\\ l's'\tilde{l}'\end{array}\right\}\left\{\begin{array}{c}L'x\tilde{L}'\\ \tilde{l}'\Lambda'l'\end{array}\right\}.$$

The analogous summation gives in the case of the jj- coupling

$$\sum_{m_{j_1'}m_{j_2'}}F^{JM_JL'M_{L'}}_{\alpha_1''\alpha_2''s'l'\Lambda'}F^{\tilde{J}\tilde{M}_J\tilde{L}'\tilde{M}_{L'}}_{\alpha_1''\alpha_2''\tilde{s}'\tilde{l}'\tilde{\Lambda}'}=$$

$$(-1)^{a+\tilde{a}}\hat{j}_1'^2\hat{j}_2'^2\hat{\tilde{\Lambda}}'^2\hat{\Lambda}'^2\hat{\tilde{J}}\hat{J}\sum_{xm_x}(-1)^{2x-2m_x}(2x+1)\{j_1'j_2'x\}\left(\begin{array}{c}L'xJ\\ M_{L'}\xi M_J\end{array}\right)\left(\begin{array}{c}\tilde{L}'x\tilde{J}\\ \tilde{M}_{L'}\xi\tilde{M}_J\end{array}\right)$$

$$\left\{\begin{array}{c}L'xJ\\ s'l'\Lambda'\end{array}\right\}\left\{\begin{array}{c}\tilde{L}'x\tilde{J}\\ \tilde{s}'\tilde{l}'\tilde{\Lambda}'\end{array}\right\}\left\{\begin{array}{c}j_1'xj_2'\\ l_1'\Lambda'l_2'\\ s_1's's_2'\end{array}\right\}\left\{\begin{array}{c}j_1'xj_2'\\ l_1'\tilde{\Lambda}'l_2'\\ s_1's's_2'\end{array}\right\}.$$

Here, $a=l_1'+l_2'+s_1'-s_2'+j_1'+j_2'+\Lambda'+2l'-2s'-2J-2L'$, $\tilde{a}=l_1'+l_2'+s_1'-s_2'+j_1'+j_2'+\tilde{\Lambda}'+2l'-2s'-2\tilde{J}-2\tilde{L}'$.

In the case of the ls- coupling it is possible to show that

$$\sum_{m_{l_1}'m_{l_2}'m_{s_1}'m_{s_2}'}\sum_{M_{L'}\tilde{M}_{L'}M_J\tilde{M}_J}F^{JM_JL'M_{L'}}_{\beta_1''\beta_2''s'l'\Lambda'}F^{\tilde{J}\tilde{M}_J\tilde{L}'\tilde{M}_{L'}}_{\beta_1''\beta_2''\tilde{s}'\tilde{l}'\ \tilde{\Lambda}'}A^{L'M_{L'};(ls)JM_JTM_T}_{\tilde{L}'\tilde{M}';(\tilde{l}\tilde{s})\tilde{J}\tilde{M}_J\tilde{T}\tilde{M}_T}(k,p,P)=$$

$$\sum_\lambda\frac{\alpha(\lambda)\beta(\lambda)}{2\lambda+1}(-1)^{\lambda+J-\tilde{J}+2L'}\{\tilde{J}J\lambda\}\{\tilde{L}'L'\lambda\}$$

$$\int q^2dqj_0(kq)j_\lambda(pq)j_\lambda(\frac{M_1}{M}Pq),$$

were $M=M_1+M_2$. In the case of the jj- coupling we have

$$\sum_{m_{j_1}'m_{j_2}'}\sum_{M_J\tilde{M}_JM_{L'}\tilde{M}_{L'}}F^{JM_JL'M_{L'}}_{\alpha_1''\alpha_2''s'l'\Lambda'}F^{\tilde{J}\tilde{M}_J\tilde{L}'\tilde{M}_{L'}}_{\alpha_1''\alpha_2''\tilde{s}'\tilde{l}'\tilde{\Lambda}'}A^{L'M_{L'};(ls)JM_JTM_T}_{\tilde{L}'\tilde{M}';(\tilde{l}\tilde{s})\tilde{J}\tilde{M}_J\tilde{T}\tilde{M}_T}(k,p,P)=$$

$$(-1)^{a+\tilde{a}}(-1)^{\tilde{l}+s+2J+\tilde{J}}\hat{j}_1'^2\hat{j}_2'^2\hat{\tilde{\Lambda}}'^2\hat{\Lambda}'^2\hat{\tilde{J}}^2\hat{J}^2\delta_{T\tilde{T}}\delta_{M_T\tilde{M}_T}\delta_{s\tilde{s}}\frac{2}{\pi}\hat{l}\hat{L}'$$

$$C^{\tilde{L}'0}_{L'0\lambda0}C^{\tilde{l}0}_{l0\lambda0}\sum_{x\lambda}(-1)^{\lambda+x+L'}(2\lambda+1)(2x+1)\{j_1'j_2'x\}\left\{\begin{array}{c}\lambda\tilde{J}J\\ sl\tilde{l}\end{array}\right\}\left\{\begin{array}{c}J\tilde{J}\lambda\\ \tilde{L}'L'x\end{array}\right\}$$

$$\left\{\begin{array}{c}L'xJ\\ s'l'\Lambda'\end{array}\right\}\left\{\begin{array}{c}\tilde{L}'x\tilde{J}\\ \tilde{s}'\tilde{l}'\tilde{\Lambda}'\end{array}\right\}\left\{\begin{array}{c}j_1'xj_2'\\ l_1'\Lambda'l_2'\\ s_1's's_2'\end{array}\right\}\left\{\begin{array}{c}j_1'xj_2'\\ l_1'\tilde{\Lambda}'l_2'\\ s_1's's_2'\end{array}\right\}$$

$$\int q^2dqj_0(kq)j_\lambda(pq)j_\lambda(\frac{M_1}{M}Pq).$$

Taking into account the formulae above, we can write the momentum disributrion in the case of the ls- coupling as follows

$$\Delta_{\overline{\beta}_1'\overline{\beta}_1'}=\sum_\lambda\sum_{N'L'\gamma'}\sum_{\tilde{N}'\tilde{L}'\tilde{\gamma}'}\sum_{lsJT}\sum_{\tilde{l}\tilde{s}\tilde{J}}M^{slJ\tilde{s}\tilde{l}\tilde{J}T}_{N'L'\gamma';\tilde{N}'\tilde{L}'\tilde{\gamma}'}(\lambda)$$

$$\int q^2dqj_0(kq)\int P^2dPj_\lambda(\frac{M_1}{M}Pq)R_{N'L'}(P)R_{\tilde{N}'\tilde{L}'}(P)\int p^2dpj_\lambda(pq)W^{\Delta NJT}_{ls,n'l's'}(p)W^{\Delta N\tilde{J}T}_{\tilde{l}s,\tilde{n}'\tilde{l}'\tilde{s}'}(p),$$

where

$$M^{slJ\tilde{s}\tilde{l}\tilde{J}T}_{N'L'\gamma';\tilde{N}'\tilde{L}'\tilde{\gamma}'}(\lambda)=(-1)^{s+\tilde{l}+\tilde{L}'+J-l'-\tilde{l}'-s'-\Lambda'}(1-(-1)^{l'+s'+T})(1-(-1)^{\tilde{l}'+\tilde{s}'+T})\hat{T}^2$$

$$\frac{2}{\pi}\delta_{s'\tilde{s}'}\delta_{\Lambda'\tilde{\Lambda}'}\delta_{s\tilde{s}}\hat{l}\hat{J}^2\hat{\tilde{J}}^2\hat{\Lambda'}^2\hat{L}'\hat{\lambda}^2C^{\tilde{L}'0}_{L'0\lambda0}C^{\tilde{l}0}_{l0\lambda0}$$

$$a^{n_1'l_1'n_2'l_2'\Lambda'}_{n'l'N'L'}a^{n_1'l_1'n_2'l_2'\tilde{\Lambda}'}_{\tilde{n}'\tilde{l}'\tilde{N}'\tilde{L}'}N^{slJ\tilde{l}\tilde{J}}_{s'l'\Lambda'\tilde{l}'}(\lambda)$$

and

$$N^{slJ\tilde{l}\tilde{J}}_{s'l'\Lambda'\tilde{l}'L'\tilde{L}'}(\lambda)=\{\tilde{J}J\lambda\}\{\tilde{L}'L'\lambda\}\left\{\begin{array}{c}\tilde{J}J\lambda\\ l'\tilde{l}'s'\end{array}\right\}\left\{\begin{array}{c}\tilde{L}'L'\lambda\\ l'\tilde{l}'\Lambda'\end{array}\right\}\left\{\begin{array}{c}\tilde{J}J\lambda\\ l\tilde{l}s\end{array}\right\},$$

$$\overline{\beta}'_i=n'_il'_is'_it'_i,\gamma'=n'l's'\Lambda'.$$

In the final expression there is the sum over the main quantum numbers, and the angular momenta dependent from the initial fixed β'_i

$$N'L's'l'n'\Lambda'\tilde{N}'\tilde{L}'\tilde{s}'\tilde{l}'\tilde{n}'\tilde{\Lambda}'.$$

Besides, there is the sum over the final angular momenta

$$slJ\tilde{s}\tilde{l}\tilde{J}\lambda T.$$

The limitations on the angular momenta follow from the 6j- symbols, which enter in expression for the constant $N^{slJ\tilde{l}\tilde{J}}_{s'l'\Lambda'\tilde{l}'L'\tilde{L}'}(\lambda)$. Fom them, the following rules of the vector addition for the angular momenta should be fulfiled: $(\tilde{J}J\lambda),(\lambda l'\tilde{l}'),(Jl's'),(s'\tilde{J}\tilde{l}')$ and $(\tilde{L}'L'\lambda),(\lambda l'\tilde{l}'),(L'l'\Lambda'),(\Lambda'\tilde{L}'\tilde{l}')$. At last, from the third the 6j- symbol we have $(\tilde{J}J\lambda),(\lambda l\tilde{l},(Jls),(s\tilde{J}\tilde{l})$. There are also limitations on the final angular momenta s,l,J following from the transition potential. The first term of the matrix element of the transition potential is different from zero only, if the triangle relations $(s's_2s_{1)},(s_11s'_1),(s_21s'_2),(s'_2s's'_1)$ are fulfiled and $l=l'$. Second term in the matrix element of transition potential is different from zero only, if the limitations on the angular momenta $(Jls),(s2s'),(l2s'),(l'Js'),(l'2l),(s_1s_2s),(s_1s'_11),(s_1s'_2s'),(s_2s'_21)$ following from the 3jm -, 6j -and 9j- symbols are fulfiled.

For the case of the jj- coupling the momentum distribution of the Δ- isobars can be written

$$\Delta_{\overline{\alpha}'_1\overline{\alpha}'_2}(k)=\sum_{N'L'n'l's'\Lambda';\tilde{N}'\tilde{L}'\tilde{n}'\tilde{l}'\tilde{s}'\tilde{\Lambda}'}\sum_{\lambda ls J\tilde{l}\tilde{J}T}M^{\lambda ls J\tilde{l}\tilde{J}T}_{N'L'n'l's'\Lambda';\tilde{N}'\tilde{L}'\tilde{n}'\tilde{l}'\tilde{s}'\tilde{\Lambda}'}\int q^2dqj_0(kq)$$

$$\int P^2dPj_\lambda(\frac{M_1}{M}Pq)R_{N'L'}(P)R_{\tilde{N}'\tilde{L}'}(P)\int p^2dpj_\lambda(pq)W^{\Delta NJT}_{ls,n'l's'}(p)W^{\Delta N\tilde{J}T}_{\tilde{l}s,\tilde{n}'\tilde{l}'\tilde{s}'}(p).$$

Here,

$$M^{\lambda ls J\tilde{l}\tilde{J}T}_{N'L'n'l's'\Lambda';\tilde{N}'\tilde{L}'\tilde{n}'\tilde{l}'\tilde{s}'\tilde{\Lambda}'}=(-1)^{\tilde{l}+s-\tilde{J}+\Lambda'+\tilde{\Lambda}'}\hat{j}_1'^2\hat{j}_2'^2\hat{\tilde{\Lambda}}'^2\hat{\Lambda}'^2\hat{\tilde{J}}^2\hat{J}^2\frac{2}{\pi}\hat{l}\hat{L}'(1-(-1)^{l'+s'+T})(1-(-1)^{\tilde{l}'+\tilde{s}'+T})\hat{T}^2$$

$$a^{n_1'l_1'n_2'l_2'\Lambda'}_{n'l'N'L'}a^{n_1'l_1'n_2'l_2'\tilde{\Lambda}'}_{\tilde{n}'\tilde{l}'\tilde{N}'\tilde{L}'}$$

$$N_{j_1'j_2'l_1'l_2's_1's_2'}(\lambda, l, s, J, \tilde{l}\tilde{J}; s'l'\Lambda'L'; \tilde{s}'\tilde{l}'\tilde{\Lambda}'\tilde{L}')$$

and

$$N_{j_1'j_2'l_1'l_2's_1's_2'}(\lambda, l, s, J, \tilde{l}\tilde{J}; s'l'\Lambda'L'; \tilde{s}'\tilde{l}'\tilde{\Lambda}'\tilde{L}') = \sum_x (-1)^{\lambda+x+L'}(2\lambda+1)(2x+1)C^{\tilde{L}'0}_{L'0\lambda 0}C^{\tilde{l}0}_{l0\lambda 0}\{j_1'j_2'x\}$$

$$\left\{ \begin{array}{c} \lambda\tilde{J}J \\ sl\tilde{l} \end{array} \right\} \left\{ \begin{array}{c} J\tilde{J}\lambda \\ \tilde{L}'L'x \end{array} \right\}$$

$$\left\{ \begin{array}{c} L'xJ \\ s'l'\Lambda' \end{array} \right\} \left\{ \begin{array}{c} \tilde{L}'x\tilde{J} \\ \tilde{s}'\tilde{l}'\tilde{\Lambda}' \end{array} \right\} \left\{ \begin{array}{c} j_1'xj_2' \\ l_1'\Lambda'l_2' \\ s_1's's_2' \end{array} \right\} \left\{ \begin{array}{c} j_1'xj_2' \\ l_1'\tilde{\Lambda}'l_2' \\ s_1's's_2' \end{array} \right\}$$

Here, $\overline{\alpha}_1' = n_i'l_i'j_i't_i'$. The limitations on the angular momentums following from the 6j- symbols are $(J\tilde{J}\lambda), (\lambda\tilde{l}l), (\tilde{J}\tilde{l}s), (sJl), (J\lambda\tilde{J}), (\tilde{J}\tilde{L}'x), (\lambda\tilde{L}'L'), (L'Jx), (L'xJ), (Js'l'), (xs'\Lambda'), (\Lambda'L'l')$, $(\tilde{L}'x\tilde{J}), (\tilde{J}\tilde{s}'\tilde{l}'), (x\tilde{s}'\tilde{\Lambda}'), (\Lambda'\tilde{L}'\tilde{l}')$.

The limitations on the angular momenta from the 9j- symbols are: $(j_1'xj_2'), (j_1'l_1's_1'), (l_1'\Lambda'l_2'), (s_1's's_2'), (x\Lambda's'), (j_2'l_2's_2'), (l_1'\tilde{\Lambda}'l_2'), (s_1'\tilde{s}'s_2'), \quad (j_1'l_1's_1'), (x\tilde{\Lambda}'\tilde{s}')$. The limitations on the angular momenta following from the transition potential coincide with limitations in the case of the ls- coupling.

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Figures

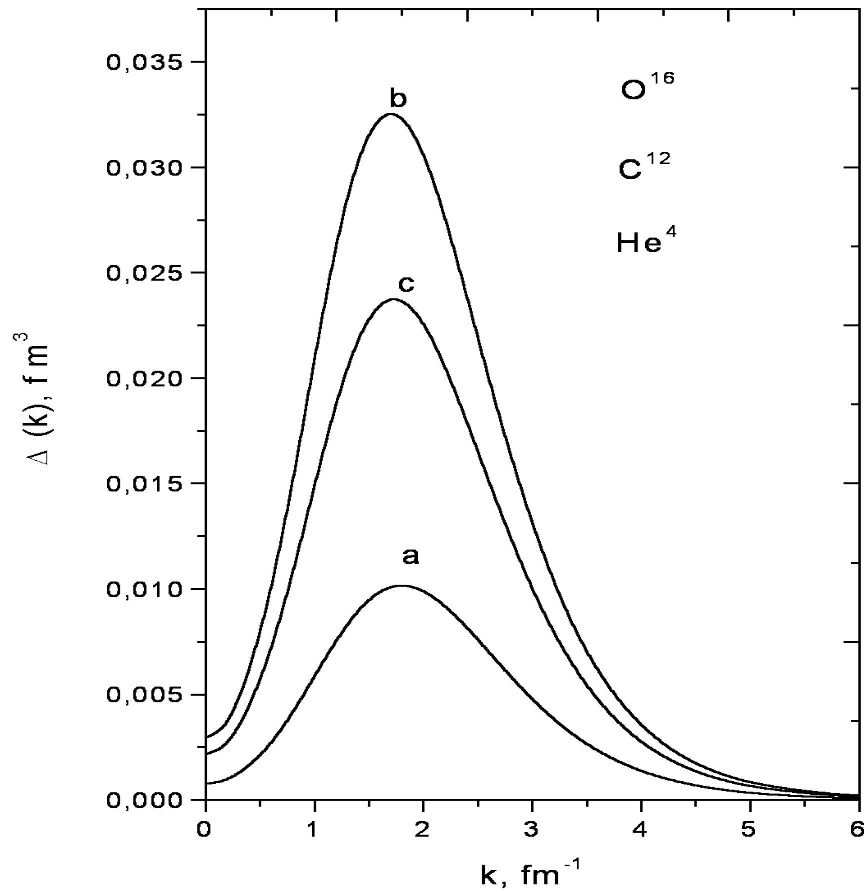

Figure 1: Momentum distribution of Δ- isobar in closed shell nuclei ^4He-a,^{16}O- b,^{12}C- c.